RADIAL–CONDUCTIVE HEAT TRANSFER OF SEMITRANSPARENT BODIES

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A method is proposed for calculating the nonsteady radiative–conductive heat transfer of semitransparent bodies in the non-one-dimensional case, on the basis of the Monte Carlo method and the finite-difference method.

Radiational–conductive heat transfer (RCH) in semitransparent bodies is of great interest at present. Although both accurate and approximate numerical methods of RCH calculation have been developed in various cases [1-3], most of these approaches are only suitable for the calculation of one-dimensional RCH. In the two- or three-dimensional case, traditional numerical methods are very cumbersome, especially when the temperature dependence of the absorption coefficients and the thermophysical parameters of the bodies must be taken into account. In that case, it is expedient to use the Monte Carlo method, in which, essentially, a random process is constructed for the given problem, and the parameters of the process are determined by calculating its statistical characteristics. Thus, in this approach, the basis for RCH calculation is the Monte Carlo method, together with the finite-difference method proposed in [4].

As a model problem, consider the nonsteady heat transfer through a glass plate bounded by semitransparent and black walls at constant temperature in the absence of scattering (Fig. 1). Such RCH may occur in the viewing windows of high-temperature processes, for example, in glass production.

The system of equations describing RCH in this case, when the absorption coefficient is independent of the wavelength and the thermophysical parameters are independent of the temperature, takes the form

$$2\mu\frac{\partial J}{\partial x}+2\sqrt{1-\mu^2}\,\frac{\partial J}{\partial y}=\alpha_R\left[\frac{1}{\pi}n^2\sigma T^4(x,y)-J\right],$$

$$\rho c_p\frac{\partial T}{\partial t}=-4\alpha_R n^2\sigma T^4(x,y)+2\pi\alpha_R\int_{-1}^{+1}J\mu d\mu+\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{1}$$

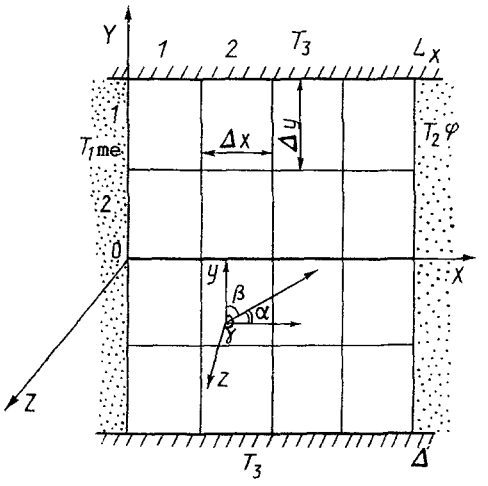

Fig. 1. Division of sample into cells.

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The boundary conditions of heat transfer are specified as conditions of the first kind

$$T(0, y, t) = T_1;\ T(L_x, y, t) = T_2;\ T(x, L_y, t) = T_3. \tag{2}$$

One initial condition is that the temperature is constant over the whole glass volume: $T(x, y, 0) = T_0$. In addition, the symmetry of heat transfer relative to the X-axis implies that: $\partial T/\partial y|_{y=0} = 0$.

For numerical solution of Eq. (1), the sample is divided into cells by lines parallel to the X and Y axes (Fig. 1), and the cooling process is divided into time intervals. Within each cell, the temperature is assumed to be constant in the given time interval.

The radiation intensity is determined by modeling the photon emission and absorption with the temperature distribution corresponding to the given time interval. The radiation intensity obtained in each cell is used to calculate the temperatures in the next time interval. Photon generation is considered for three types of sources:

1) surface sources

$$S^k = n^2\sigma (T_3^k)^4 \Delta x; \tag{3}$$

2) the media surrounding the semitransparent boundaries

$$S_m^k = \sigma (T_m^k)^4 \Delta y; \tag{4}$$

3) volume sources

$$S_{i,j}^k = 4n^2\sigma (T_{i,j}^k)^4 \Delta x \Delta y \alpha_R. \tag{5}$$

The number of particles emitted is determined from the relation

$$N_{i,j}^k = E\left[\frac{NS_{i,j}^k}{\sum\limits_{m,l} S_{m,l}^k + \sum S_{i,j}^k}\right]. \tag{6}$$

Each particle generated is identified with a photon beam of energy

$$G_{i,j}^k = S_{i,j}^k \Delta t / N_{i,j}^k. \tag{7}$$

Since $C\Delta t \gg L$, all the particles generated are either absorbed or leave the sample at the end of each time interval. As well as the particle energy, the particle coordinates are calculated from relations analogous to those in [4]:

1) for surface particles

$$x_0 = 0, \qquad x_0 = L_x, \qquad x_0 = (i - 1 + R)\Delta x, \tag{8}$$

$$y_0 = (j - 1 + R)\Delta y, \quad y_0 = (j - 1 + R)\Delta y, \quad y_0 = L_y;$$

2) for volume particles

$$x_0 = (i - 1 + R)\Delta x, \quad y_0 = (j - 1 + R)\Delta y. \tag{9}$$

Since the calculations assume a Cartesian coordinate system, the directional cosines given by the relations in [5] must be used to specify the direction of flight of the particle

$$\cos\gamma = \frac{R_1}{\sqrt{R_1^2 + R_2^2 + R_3^2}}$$

$$\text{when } R_1^2 + R_2^2 + R_3^2 \leqslant 1. \tag{10}$$

$$\cos\beta = \frac{R_2}{\sqrt{R_1^2 + R_2^2 + R_3^2}}$$

For surface particles with coordinates $x = 0$ and $x = L$, taking account of refraction at the boundaries, the directional cosines are

$$\cos\gamma' = (1/n)\sqrt{n^2 + \cos^2\gamma - 1}, \quad \cos\beta' = \cos\beta/n. \tag{11}$$

Then the distance covered by the particle up to absorption is calculated from the expression [6]

$$d = |\ln R|/\alpha_R \tag{12}$$

and the coordinates of the cell in which absorption occurs are

$$i' = E[(x_0\cos\gamma + d)/(\Delta x\cos\gamma)] + 1, \quad j' = E[(y_0\cos\beta + d)/(\Delta y\cos\beta)] + 1. \tag{13}$$

If $j' < 0$ or $j' > h_y$, the particle is assumed to leave the sample. If $i' < 0$ or $i' > h_x$ and, in addition, the angle between the direction of flight of the particle and the perpendicular to the plate boundary is no greater than the angle of total internal reflection, i.e., $n\sqrt{1-\cos^2\gamma} < 1$, the particle is assumed to undergo total internal reflection. The coordinate along the X-axis of this plate is calculated from the recurrence relations

$$i_s' = \begin{cases} E\left[(L_x - d_{s-1}/\cos\gamma)/\Delta x\right] + 1 & \text{when } i_{s-1}' > 0, \\ E\left[d_{s-1}/(\Delta x\cos\gamma)\right] + 1 & \text{when } i_{s-1}' < 0 \end{cases} \tag{14}$$

until $0 < i_s' < h_x$.

After considering the trajectories of all the particles, the energy balance of each cell is determined

$$\Delta G_{i,j}^k = N_{i,j}^k G_{i,j}^k - \sum_{p=1}^{q} G_{i,j,p}^k. \tag{15}$$

The temperature of each cell is determined from the system of equations consisting of the energy equations from Eq. (1) in dimensionless and finite-difference form

$$\theta_{i,j}^{k+1/2} - \theta_{i,j}^k = \frac{\sqrt{\mathrm{Fo}_x\mathrm{Fo}_y}\,\Delta\tau_x\Delta\tau_y}{P}\Delta Q_{i,j}^k + \\ + \mathrm{Fo}_x\left(\theta_{i-1,j}^{k+1/2} - 2\theta_{i,j}^{k+1/2} + \theta_{i+1,j}^{k+1/2}\right) + \mathrm{Fo}_y\left(\theta_{i,j-1}^{k} - 2\theta_{i,j}^{k} + \theta_{i,j+1}^{k}\right),$$

$$\theta_{i,j}^{k+1} - \theta_{i,j}^{k+1/2} = \frac{\sqrt{\mathrm{Fo}_x\mathrm{Fo}_y}\,\Delta\tau_x\Delta\tau_y}{P}\Delta Q_{i,j}^{k+1/2} + \\ + \mathrm{Fo}_x\left(\theta_{i-1,j}^{k+1/2} - 2\theta_{i,j}^{k+1/2} + \theta_{i+1,j}^{k+1/2}\right) + \mathrm{Fo}_y\left(\theta_{i,j-1}^{k+1} - 2\theta_{i,j}^{k+1} + \theta_{i,j+1}^{k+1}\right). \tag{16}$$

To determine the temperatures $\theta_{i,j}^{k+1/2}$ and $\theta_{i,j}^{k+1}$, Eq. (16) is solved numerically in each integer and semi-integer time step by trial and error.

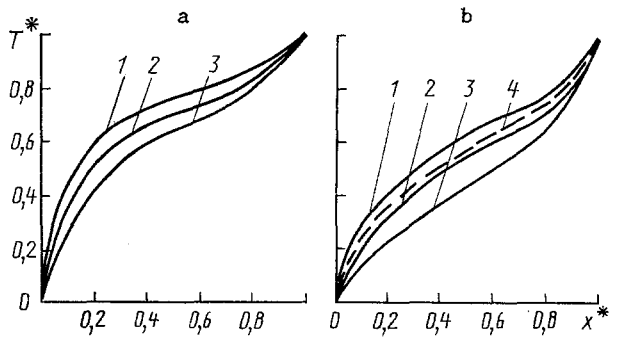

Fig. 2. Temperature distribution in sample along X-axis: a) Fo = 0.01; b) Fo = ∞; 1) y = 0; 2) 0.35; 3) 0.5; 4) 0 (data of [7]).

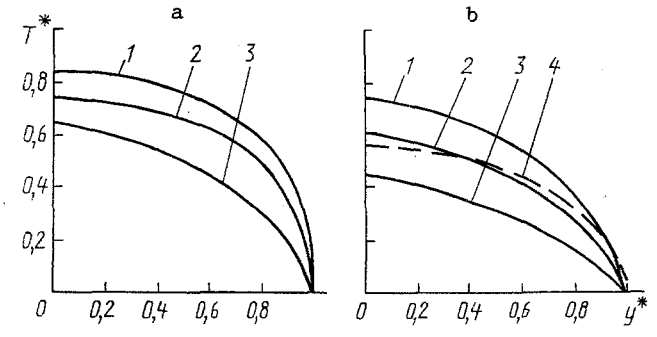

Fig. 3. Temperature distribution in sample along Y-axis: a) Fo = 0.01; b) Fo = ∞; 1) x = 0.75; 2) 0.5; 3) 0.25; 4) 0.5 (data of [7]).

This method is used to calculate the temperature field of a glass plate with the parameter values $\tau_x = 1$, $\tau_y = 2$, $P = 0.04$, $\theta_1 = 0.5$, $\theta_2 = 1$, $\theta_3 = 0.5$. The results are shown in Figs. 2 and 3.

Comparison of Figs. 2 and 3 with the temperature distribution in the steady case (Fo = ∞) obtained by the approximate method of [7] shows that the maximum discrepancy is around 10%. Comparison of the temperature field obtained with the results of one-dimensional calculation for conditions corresponding to the middle of the plate shows that the distributions are similar. However, the temperature is somewhat lower in the two-dimensional case, which is explained by heat extraction from the lateral surfaces.

The error in calculating the temperature fields is determined by two methods: 1) that described in [4]; 2) numerical experiment. In the latter case, the surface temperature is specified as equal to the initial temperature of the sample. Then the RCH is calculated over several tens of time cycles, and the mean square deviation of the temperature of each cell from the initial values is determined. Calculations show that, with 100,000 particles in each interval and a confidence level of 0.99, the maximum confidence interval in both cases is no more than $0.01T_0$.

The results obtained indicate that the method proposed may be used for complex (two- and three-dimensional) RCH calculations and for the case when the complex interaction of the boundary with radiation must be taken into account.

NOTATION

T, absolute temperature of glass; T_1, T_2, temperature of the medium on both sides of the window; T_3, temperature of wall in contact with glass; i, j, k, subscripts characterizing the division of the sample with respect to the X and Y axes and the time, respectively; m = 1, 2, 3, subscript denoting one of the media or the wall; ℓ, analog of i, j for surface particles; ρ, c_p, λ, density, specific heat, and thermal conductivity of glass; σ, Stefan–Boltzmann constant; n, refractive index of glass; X, Y, coordinate axes; Δx, Δy, coordinate steps; μ, cosine of the angle between the direction of the radiation and the X axis; t, time; $S_{m,\ell}k$, $S_{i,j}k$, radiation flux density of surfaces and each cell in the k-th time interval; E[...], integer part of number; N, total number of particles; $N_{i,j}^k$, number of surface or volume particles generated in each time interval; Δt, time interval; C, velocity of light; L, characteristic dimension of sample; L_x, L_y, dimensions of sample along the X and Y axes; d, distance covered by the particle before absorption; R, random number uniformly distributed in the interval 0-1; R_z (z = 1, 2, 3), three successive random numbers equal to 1 − 2R; γ, β, angles between the particle trajectories and the X and Y axes; $h_x = L_x/\Delta x$, $h_y = L_y/\Delta y$, number of intervals along the X and Y axes; $\sum_{p=1}^{q} G_{i,j,p}^k$, total energy of the radiation absorbed by the cell; $\theta_{i,j}^k$, dimensionless temperature of cell; $\theta_3 = T_3/T_0$, dimensionless temperature of boundary surfaces; $\theta_1 = T_1/T_0$, $\theta_2 = T_2/T_0$, dimensionless temperatures of media 1 and 2; $\Delta Q_{i,j}^k = -\Delta G_{i,j}^k/(n^2\sigma T_0^4 \Delta x \Delta y \alpha_R)$ dimensionless analog of $\Delta G_{i,j}^k$; Fo_x, Fo_y; Fourier numbers along the X and Y axes; $\Delta\tau_x = \alpha_R \Delta x$, $\Delta\tau_y = \alpha_R \Delta y$ optical lengths of cell; $\tau_x = \alpha_R L_x$, $\tau_y = \alpha_R L_y$ optical lengths of plate along the X and Y axes; $P = (\lambda\alpha_R)/(n^2\sigma T_0^3)$ radiational–conductive parameter; $x^* = x/L_x$, $y^* = y/L_y$, dimensionless coordinates; $T^* = (T - T_3)/(T_0 - T_3)$, relative temperature.

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